Polyhedral Extraction Tool (http://freecode.com/projects/libpet)

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Polyhedral Program Analysis and Transformation

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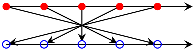

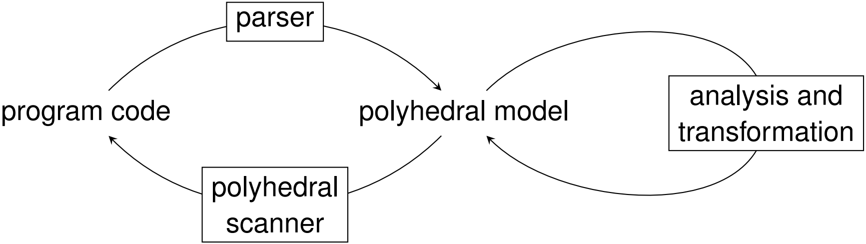


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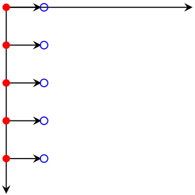

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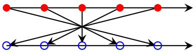

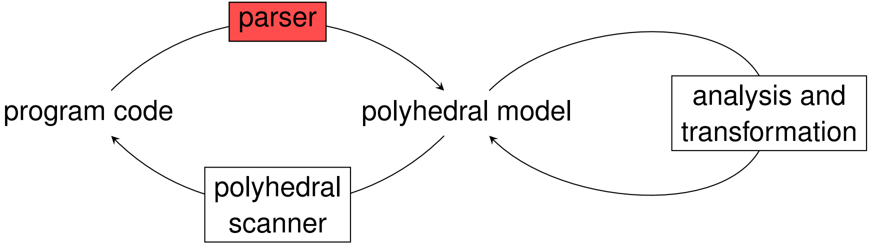


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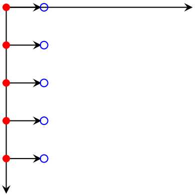


Basic Requirements

- Open source
- C99
  - iterator declarations

    ```
    for (int i = 0; i < N; ++i)
    ```

  - variable length arrays
    ⇒ parametric analysis
    ⇒ especially when arrays need to be linearized (e.g., CUDA)
- AST-level
  ⇒ source-to-source

Polyhedral Parsers

Cosy

LLVM/Polly

WRaP-IT gcc/graphite

clan

CHiLL

LooPo pers

ROSE/PolyOpt

IBM/XL insieme

ROSE/PolyherdalModel

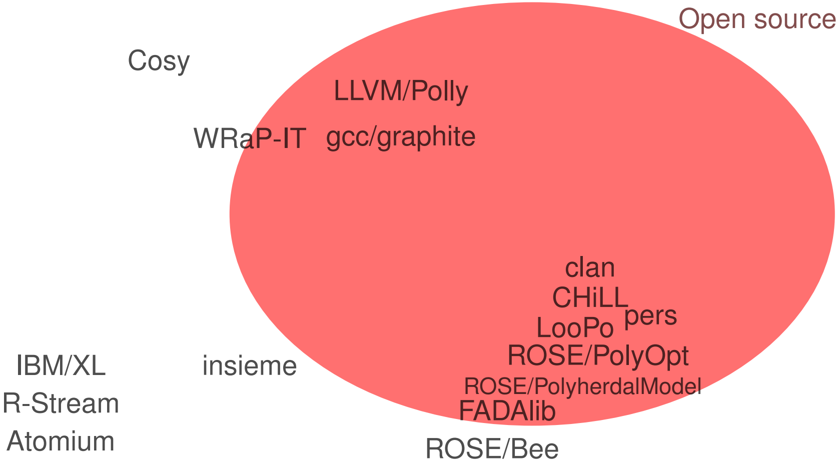
R-Stream

FADAlib

Atomium

ROSE/Bee

Polyhedral Parsers



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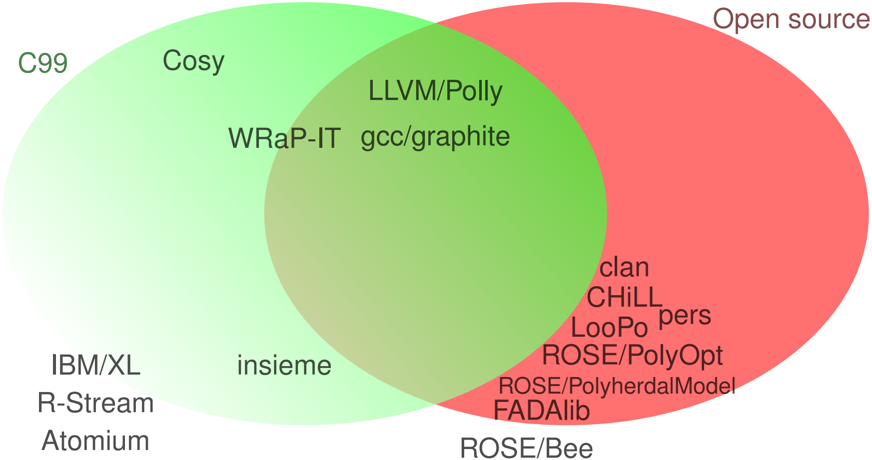

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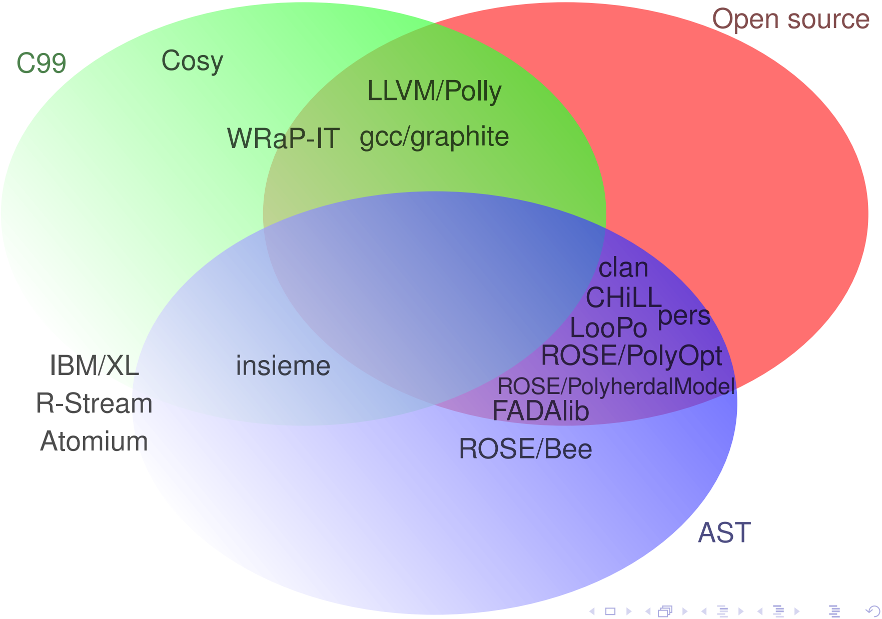

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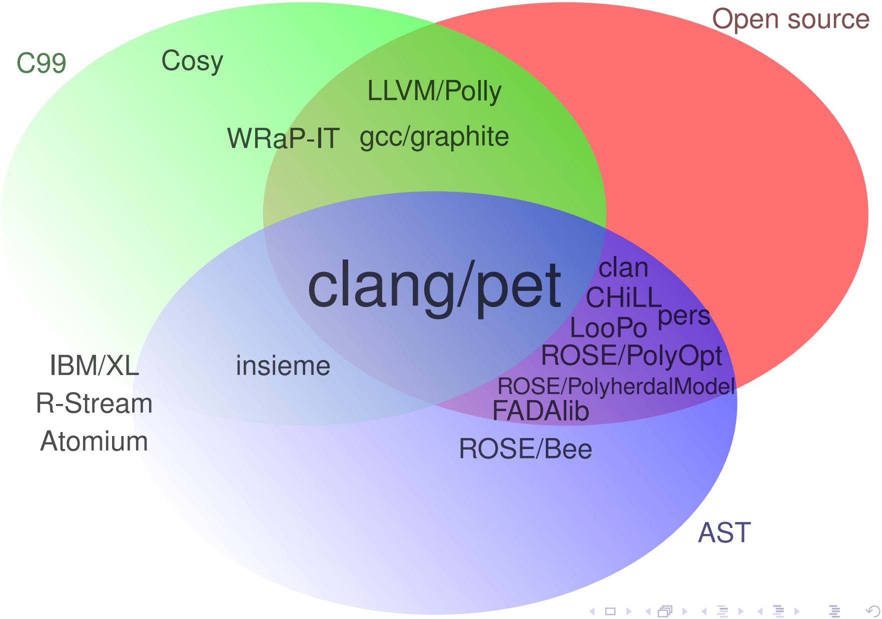

Additional Requirements

- avoid arbitrary restrictions
- support features of both `clan` and `pers`

Before, we used

- `clan`
  - scops delimited by pragmas
  - used by PPCG: source-to-source compilers
    target (currently): CUDA
- `pers` (SUIF)
  - scops autodetected
  - used by equivalence checker
    - CLooG outputs
    - data dependent constructs
    - array slices
  - used for derivation of polyhedral process networks
    - infinite time loop

Avoid Arbitrary Restrictions

Conditions and Index Expressions

Piecewise quasi-affine partial functions ($\approx$ quasts) used to represent

- conditions ($\Rightarrow$ yes, no, undefined)
- index expressions

(during construction)

May involve

- +, – (both unary and binary)
- * (at least one argument is piecewise constant)
- /, % (second argument is constant)
  `a / b` is constructed as `a >= 0 ? floord(a,b) : ceild(a,b)`
- `?:`
- &&, `||`, `!`
- `<`, `<=`, `>`, `>=`, `==`, `!=`

Avoid Arbitrary Restrictions

Loops

```
for (i = init(n); condition(n,i); i += v)
```

- unique induction variable (may be declared)
- increment: `i -= -v`, `i = i + v`, `++i` or `--i`
- **any** static piecewise quasi-affine condition
  $\Rightarrow$ needs to be satisfied for **all** iterations
  Let
  $$D = \{ i \mid \exists \alpha : \alpha \geq 0 \wedge i = \mathtt{init}(\mathbf{n}) + \alpha v \}$$
  $$C = \{ i \mid \mathtt{condition}(\mathbf{n}, i) \}$$
  Iteration domain (for $v > 0$):
  $$D \setminus (\{ i' \to i \mid i' \leq i \}(D \setminus C)) .$$

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Infinite loops

- `for (;;)`
- `while (1)`

Context and Array Slices

Context describes assumptions on the parameters

Excludes

- values outside of parameter representation
- values that lead to negative array sizes
- values that necessarily lead to overflows

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Access to array row

```
int A[M][N];
f(A[4]);
```

⇒ access relation: `[N, M] -> { S_0[] -> A[4, o1] }`

Parsing CLooG output

```
for (c1=ceild(n,3);c1<=floord(2*n,3);c1++) {
  for (c2=0;c2<=n-1;c2++) {
    for (j=max(1,3*c1-n);j<=min(n,3*c1-n+4);j++) {
      p = max(ceild(3*c1-j,3),ceild(n-2,3));
      if (p <= min(floord(n,3),floord(3*c1-j+2,3))) {
        S2(c2+1,j,0,p,c1-p);
      }
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  }
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```

- forward substitution
- special treatment of `floord` and `ceild`
- special treatment of `min` and `max`

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Data Dependent Accesses and Conditions

Data dependent access

```
A[i + 1 + in2[i]]
```

- values of nested accesses are encoded in domain of access relation
- domain of outer access relation is itself a (wrapped) map
  - domain of wrapped map is the iteration domain
  - range of wrapped map are the values of the nested accesses

  ```
  { [S_4[i] -> [i1]] -> A[1 + i + i1] }
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- list of nested access relation is maintained separately

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  { S_4[i] -> in2[i] }
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Data dependent conditions are handled similary
⇒ statement domain is wrapped map

Equivalence Checking Example

```
for (i = 0; i < M; ++i) {
  m = i+1;
  for (j = 0; j < N; ++j)
    m = g(h(m), in1[i][j]);
  compute_row(h(m), A[M-i-1]);
}
A[5][6] = 0;
for (i = 0; i < M - 2; ++i)
  out[i] = f(A[M-i-2-in2[i]]);

for (i = 0; i < M; ++i) {
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  if (i >= 2)
    out[i-2]=f(B[i-1+in2[i-2]]);
}
```

Are the two programs on the left equivalent?

$\Rightarrow$ Same output when given same input

Yes, except at $[M-8, M-6]$ (when value of in2 in [-1,1])

Assumptions

- no pointers
- no recursion
- functions called are pure
- static control flow
- quasi-affine loop bounds
- quasi-affine conditions
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Support for unsigned integers

In C, unsigned integers undergo wrapping

- unsigned expressions are reduced modulo `UINT_MAX + 1`
  ⇒ `clang` tells us which expressions are unsigned + size
- use virtual iterator for loops with unsigned iterator
  ⇒ loop condition is composed with wrapping
  ⇒ schedule domain intersected with iteration domain
  ⇒ wrapping applied to domain and schedule

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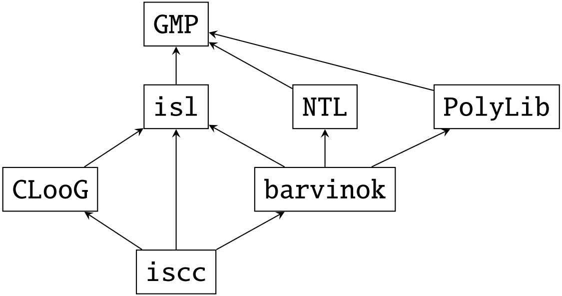
```
for (unsigned char k=252; (k%9) <= 5; ++k)
        S:;

domain: '{ S[k] : exists (e0 = [(507 - k)/256]:
        k >= 0 and k <= 255 and 256e0 >= 252 - k
        and 256e0 <= 261 - k) }'
schedule: '{ S[k] -> [0, o1] :
        exists (e0 = [(-k + o1)/256]:
        256e0 = -k + o1 and o1 >= 252 and
        k <= 255 and k >= 0 and o1 <= 261) }'
```

Integration into iscc

iscc: interactive environment
isl: manipulates parametric affine sets and relations
barvinok: counts elements in parametric affine sets and relations
CLooG: generates code to scan elements in parametric affine sets



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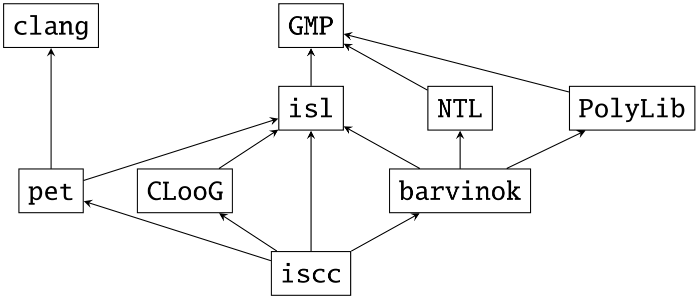
pet: extracts polyhedral model



Maximal Number of Live Memory elements

```
for (i = 0; i < N; ++i)
S1:      t[i] = f(a[i]);
for (i = 0; i < N; ++i)
S2:      b[i] = g(t[N-i-1]);
```

```
D := [N] -> { S1[i] : 0 <= i < N; S2[i] : 0 <= i < N };
R := [N] -> { S1[i] -> a[i]; S2[i] -> t[N-i-1] } * D;
W := { S1[i] -> t[i]; S2[i] -> b[i] } * D;
S := { S1[i] -> [0,i]; S2[i] -> [1,i] } * D;
Dep := (last W before R under S)[0];
LR := (lexmax (Dep . S)) . S^-1;
LLT := S << S; LGE := S >>= S;
After_Write := domain_map(LR) . LLT;
Before_Read := range_map(LR) . LGE;
N_Live := card ((After_Write * Before_Read)^-1);
ub N_Live;
```

Result:

```
([N] -> { max(N) : N >= 2; max(N) : N = 1 }, True)
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```

```
M := parse_file("live.c");
D := M[0]; W := M[1]; R := M[2]; S:= M[3] * D;
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