Multifor for Multicore

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  - programming languages:
    - many new languages are or have been proposed, many have disappeared or are going to disappear
    - current successful languages can be extended

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  - code optimization and parallelization:
    - standard developers have to be raised to
      20 years ago experienced programmers
    - as they learned the use of functions, recursion, object programming, ...
      they should learn data layout optimization, simple loop transformations, mapping of iteration/data domains, ...
    - but without being forced to
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    - but without being forced to
      (optional constructs vs specific languages)
  - hardware/software support mechanisms for parallel programming cannot solve all parallel programming issues,
    when they do not even add some more problems
    (TM, VM, etc.)

Why a Multifor construct?

- The Polytope Model
  - most of its features are hidden to developers (automatic parallelization)
  - polyhedral transformations result often in (efficient but) unreadable code
  - the model's scope is not limited to a sequence of loop nests, and can be applied incrementally
  - *polyhedral programming* can promote the model and improve its efficiency

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  - most of its features are hidden to developers (automatic parallelization)
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  - the model's scope is not limited to a sequence of loop nests, and can be applied incrementally
  - *polyhedral programming* can promote the model and improve its efficiency
- Multifor
  - a polyhedral programming control structure, providing a polyhedral view of the computation
  - facilitates the expression of some task parallelism, dataflow and MapReduce schemes
  - allows developers to express some loop fusion, mapping of domains, data reuse, ...

Syntax and semantics

$$
\begin{aligned}
\textbf{multifor}\ (\quad & index_1 = expr, [index_2 = expr, ...]\ ; \\
& index_1 < expr, [index_2 < expr, ...]\ ; \\
& index_1 += cst, [index_2 += cst, ...]\ ; \\
& grain_1, [grain_2, ...]\ ; \\
& offset_1, [offset_2, ...]\ )\ \ \{ \\
& prefix : \{statements\} \\
\}
\end{aligned}
$$

where

- *expr*: affine arithmetic expressions on enclosing loop indices
- *cst*, *grain* and *offset*: integer constants
- $grain \geq 1, offset \geq 0$
- *prefix*: positive integer associating statements to their corresponding for-loop

Syntax and semantics

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- Every iteration domain is mapped on a same referential iteration domain, according its grain and offset
  - referential domain: union of the for-loop domains, dilated and shifted following their respective grain and offset
  - grain: frequency in which the loop is run, gcd of the grains of the overlapping for-loops per sub-domain (compression factor)
  - offset: gap between the first iteration of the referential domain and the first iteration of the loop

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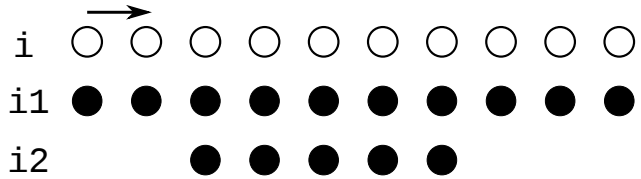
  - offset: gap between the first iteration of the referential domain and the first iteration of the loop
- On overlapping for-loops iteration domains, respective iterations are run in any interleaved fashion or in parallel.

Examples: one multifor-loop

offset

multifor $(i_1 = 0, i_2 = 10; i_1 < 10, i_2 < 15; i_1 ++, i_2 ++; 1, 1; 0, 2)$



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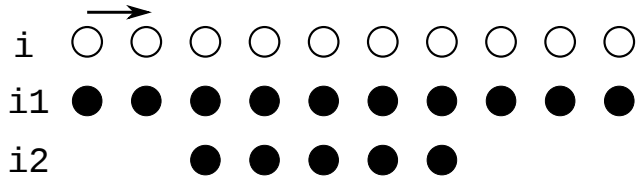
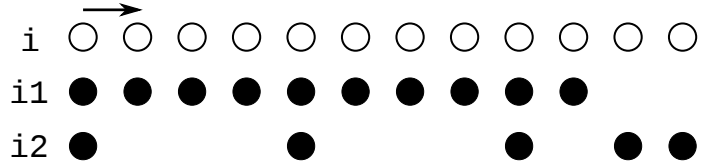


grain + compression

multifor $(i_1 = 0, i_2 = 10; i_1 < 10, i_2 < 15; i_1 ++, i_2 ++; 1, 4; 0, 0)$



Nested multifor-loops

multifor ($index_1 = expr, index_2 = expr;$
$index_1 < expr, index_2 < expr;$
$index_1 += cst, index_2 += cst;$
$grain_1, grain_2; offset_1, offset_2$) {
$prefix : \{statements\}$

  multifor ($index_3 = expr, index_4 = expr;$
  $index_3 < expr, index_4 < expr;$
  $index_3 += cst, index_4 += cst;$
  $grain_3, grain_4; offset_3, offset_4$) {
  $prefix : \{statements\}$

  }

$prefix : \{statements\}$

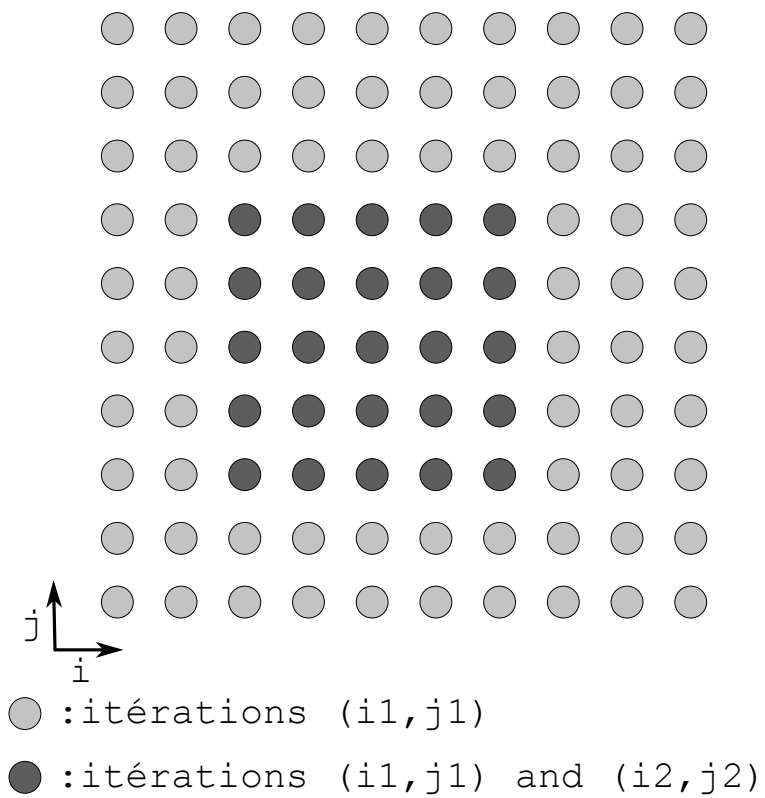
}

- behaves as 2 for-loop nests ($index_1, index_3$) and ($index_2, index_4$)
- the bounds are affine functions of the enclosing loop indices of the same for-loop

Examples: nested multifor-loops

offset

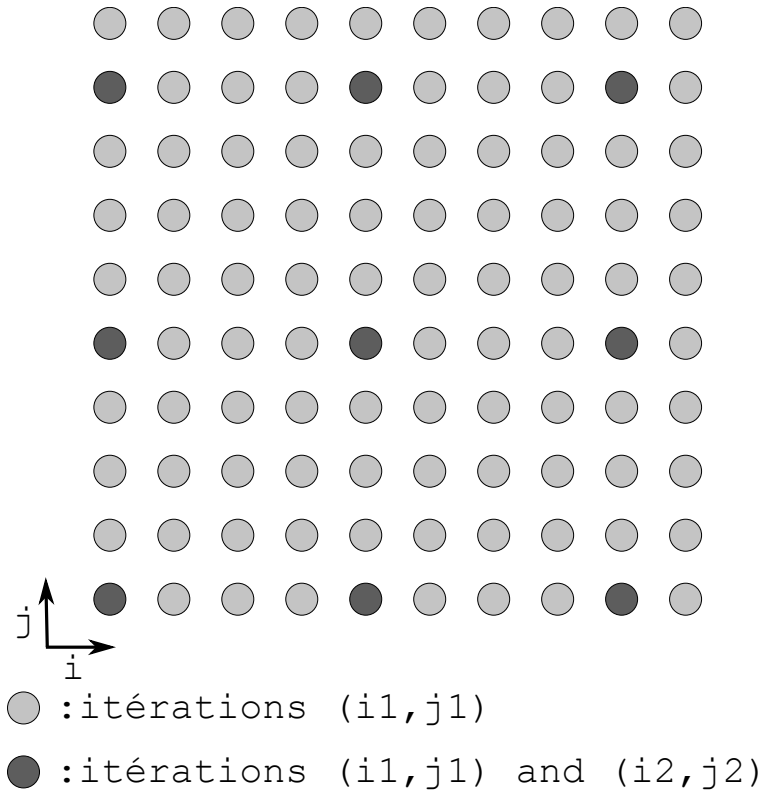
multifor $(i_1 = 0, i_2 = 0; i_1 < 10, i_2 < 5; i_1 + +, i_2 + +; 1, 1; 0, 2)$
multifor $(j_1 = 0, j_2 = 0; j_1 < 10, j_2 < 5; j_1 + +, j_2 + +; 1, 1; 0, 2)$



Examples: nested multifor-loops

grain

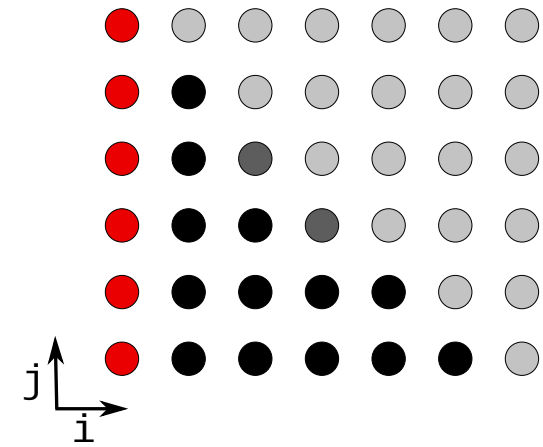
multifor $(i_1 = 0, i_2 = 0; i_1 < 10, i_2 < 3; i_1 + +, i_2 + +; 1, 4; 0, 0)$
multifor $(j_1 = 0, j_2 = 0; j_1 < 10, j_2 < 3; j_1 + +, j_2 + +; 1, 4; 0, 0)$



Examples: nested multifor-loops

affine bound + offset

multifor $(i_1 = 0, i_2 = 0; i_1 < 6, i_2 < 6; i_1 + +, i_2 + +; 1, 1; 0, 1)$
multifor $(j_1 = 0, j_2 = 0; j_1 < 6 - i_1, j_2 < 6; j_1 + +, j_2 + +; 1, 1; 0, 0)$



● :itérations (i1,j1)

● :itérations (i1,j1) and (i2,j2)

● :itérations (i2,j2)

Multifor-loop parallelization and transformation

- Parallelization opportunities:
  - Running each for-loop as a separated thread
  - Parallelizing each for-loop in an OpenMP fashion
  - Parallelizing simultaneously in both ways
- Polyhedral transformations:
  - Of each for-loop
  - With a global view regarding their interactions (referential domain)

Another way of writing loop nests

- Imperfect nests:

for $(i = 0; i < 10; i++)$
  $inst\_block_1$
  for $(j = 0; j < 10; j++)$
    $inst\_block_2$

or

multifor $(i_1 = 0, i_2 = 0; i_1 < 10, i_2 < 10; i_1++, i_2++; 1, 1; 0, 0)$
  multifor $(j_1 = 0, j_2 = 0; j_1 < 1, j_2 < 10; j_1++, j_2++; 1, 1; 0, 1)$
    $0 : inst\_block_1$
    $1 : inst\_block_2$

Another way of writing loop nests

- Re-scheduling some statements, e.g. for data locality:

for $(i = 0; i < 100; i++)$
  for $(j = 0; j < 100; j++)$
    $b += a[i][j] + 1;$
    $c += a[i+1][j+1] + 2;$

transformed to:

multifor $(i_1 = 0, i_2 = 0; i_1 < 100, i_2 < 100; i_1++, i_2++; 1, 1; 1, 0)$
  multifor $(j_1 = 0, j_2 = 0; j_1 < 100, j_2 < 100; j_1++, j_2++; 1, 1; 1, 0)$
    $0 : b += a[i][j] + 1;$
    $1 : c += a[i+1][j+1] + 2;$

Another way of writing loop nests

- Tiling:

for $(it = 0; it < N; it += tsize1)$
  for $(jt = 0; jt < N; jt += tsize2)$
    for $(i = it; i < it + tsize1; i++)$
      for $(j = jt; j < jt + tsize2; j++)$
        $inst\_block$

or:

multifor $([N/tsize1]\ i = [0, tsize1]; i < i + tsize1; i++;$
  $[N/tsize1]\ 1; [0, tsize1])$
  multifor $([N/tsize2]\ j = [0, tsize2]; j < j + tsize2; j++;$
    $[N/tsize2]\ 1; [N/tsize2]\ 0)$
    $* : inst\_block$

- Requires some extensions:
  - $[n]\ i$: n indices $i_1, i_2, ..., i_p$
  - $[a, b]$: n values $a, a + b, a + 2b, a + 3b, ...$
  - $[n]\ m$: $m, m, m, ...$ (n times) ; $*$: every nest executes

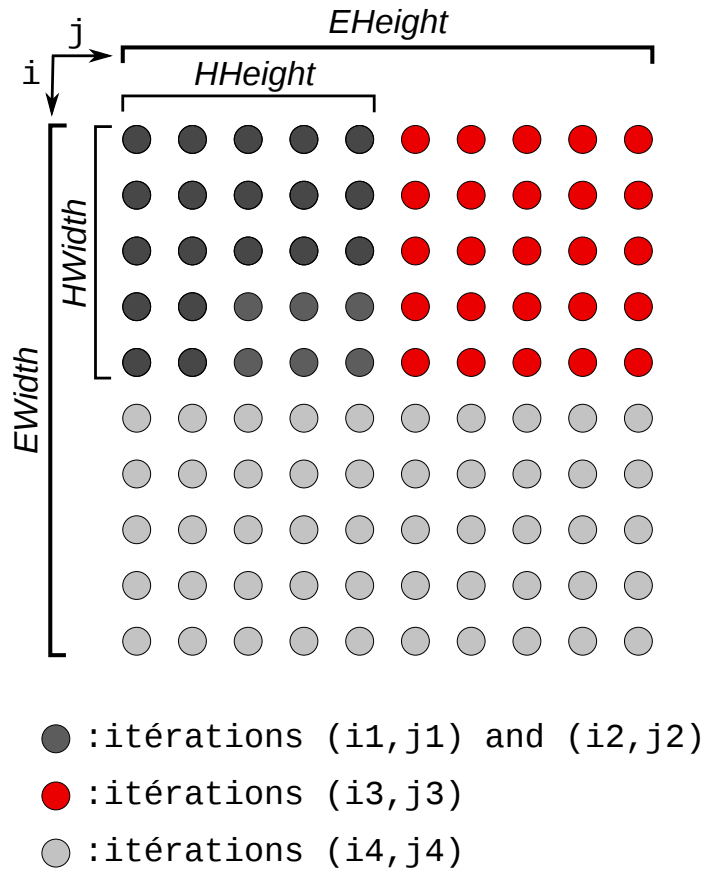
Some *real* examples

- Steganography: decoding phase where a $(HWidth \times HHeight)$ image is hidden in a $(EWidth \times EHeight)$ image

multifor $(i_1 = 0, i_2 = 0; i_3 = 0, i_4 = HWidth; i_1 < HWidth,$
$i_2 < HWidth, i_3 < HWidth, i_4 < EWidth;$
$i_1 ++, i_2 ++, i_3 ++, i_4 ++; 1,1,1,1; 0,0,0,0)$
multifor $(j_1 = 0, j_2 = 0, j_3 = HHeight, j_4 = 0; j_1 < HHeight,$
$j_2 < HHeight, j_3 < EHeight, j_4 < EHeight;$
$j_1 ++, j_2 ++, j_3 ++, j_4 ++; 1,1,1,1; 0,0,0,0)$
{
0 : // Retrieve the hidden image
$* HImage(i_1, j_1) = decode\_hidden(i_1, j_1);$
1 : // Retrieve the enclosing image
$* MImage(i_2, j_2) = decode\_main(i_2, j_2);$
$[2,3]$: // Retrieve the enclosing image
$* MImage([i_3, i_4], [j_3, j_4]) = *EImage([i_3, i_4], [j_3, j_4]);$
}

Some *real* examples

- Steganography: multifor-loop nest scan of the images



Some *real* examples

- Red-Black Gauss-Seidel: traditional code

```
// Red phase
for (i = 1; i < N - 1; i++)
 for (j = 1; j < N - 1; j++)
  if ((i + j) % 2 == 0)
    u[i][j] = f(u[i][j + 1], u[i][j - 1], u[i - 1][j], u[i + 1][j]);

// Black phase
for (i = 1; i < N - 1; i++)
 for (j = 1; j < N - 1; j++)
  if ((i + j) % 2 == 1)
    u[i][j] = f(u[i][j + 1], u[i][j - 1], u[i - 1][j], u[i + 1][j]);
```

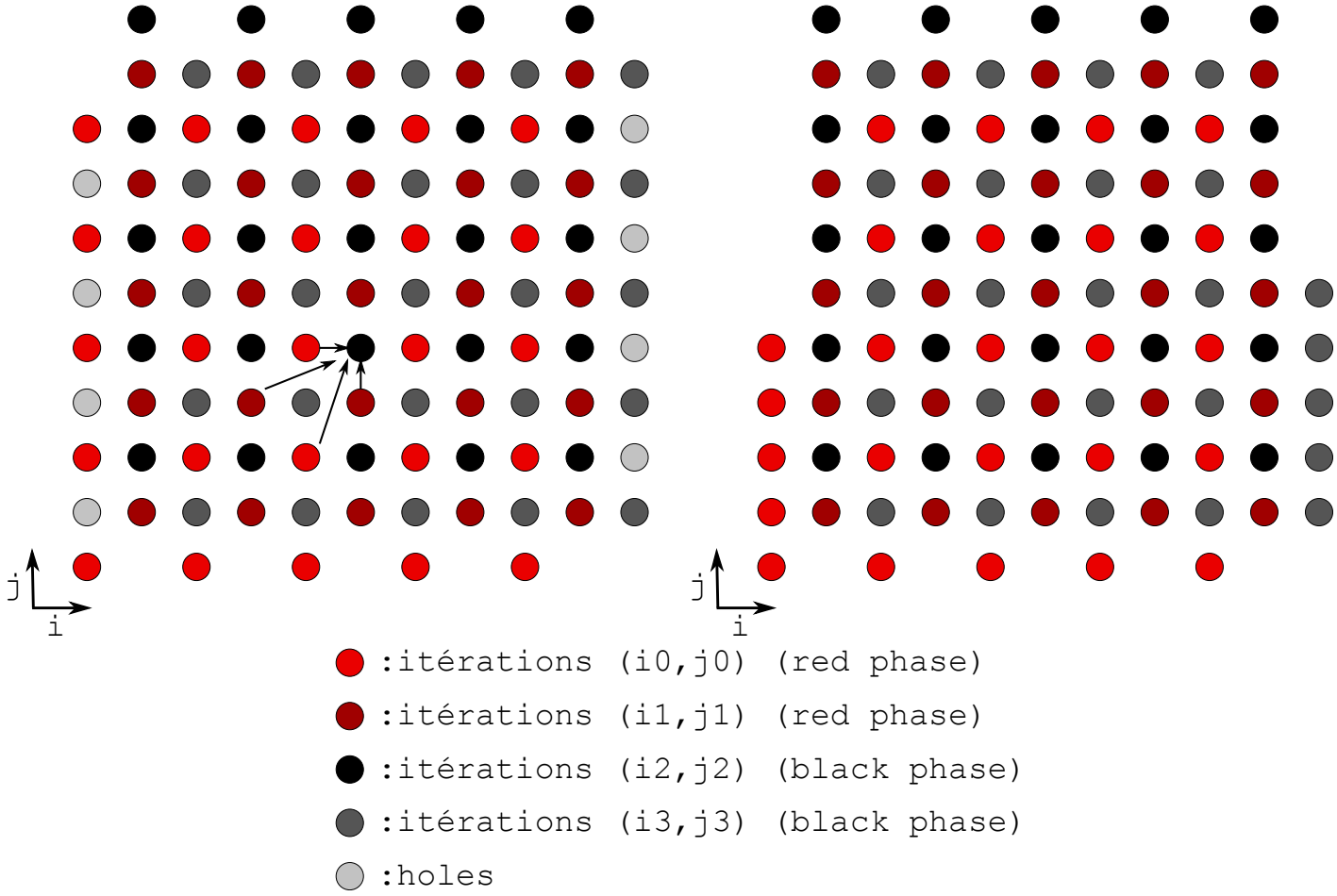
Some *real* examples

- Red-Black Gauss-Seidel: multifor code

$$
\begin{aligned}
&\textbf{multifor}\ (i_0 = 1, i_1 = 2, i_2 = 1, i_3 = 2; i_0 < N-1, i_1 < N-1,\\
&\qquad i_2 < N-1, i_3 < N-1; i_0 += 2, i_1 += 2, i_2 += 2,\\
&\qquad i_3 += 2; 2,2,2,2; 0,1,1,2)\\
&\textbf{multifor}\ (j_0 = 1, j_1 = 2, j_2 = 2, j_3 = 1; j_0 < N-1, j_1 < N-1,\\
&\qquad j_2 < N-1, j_3 < N-1; j_0 += 2, j_1 += 2, j_2 += 2,\\
&\qquad j_3 += 2; 2,2,2,2; 0,1,2,1)\ \{\\
&\quad 0 : u[i_0][j_0] =\\
&\qquad f(u[i_0][j_0+1], u[i_0][j_0-1], u[i_0-1][j_0], u[i_0+1][j_0]);\\
&\quad 1 : u[i_1][j_1] =\\
&\qquad f(u[i_1][j_1+1], u[i_1][j_1-1], u[i_1-1][j_1], u[i_1+1][j_1]);\\
&\quad 2 : u[i_2][j_2] =\\
&\qquad f(u[i_2][j_2+1], u[i_2][j_2-1], u[i_2-1][j_2], u[i_2+1][j_2]);\\
&\quad 3 : u[i_3][j_3] =\\
&\qquad f(u[i_3][j_3+1], u[i_3][j_3-1], u[i_3-1][j_3], u[i_3+1][j_3]);\\
&\}
\end{aligned}
$$

Some *real* examples

- Red-Black Gauss-Seidel: multifor referential domain



Some *real* examples

- Red-Black Gauss-Seidel: possible generated for-loop code

for $(j = 1; j < N-1; j+=2)$
$u[i][j] = f(u[i][j+1], u[i][j-1], u[i-1][j], u[i+1][j]);$
for $(i = 2; i < N-2; i+=2)$ {
for $(j = 2; j < N-1; j+=2)$ {
$u[i][j] = f(u[i][j+1], u[i][j-1], u[i-1][j], u[i+1][j]);$
$u[i][j+1] = f(u[i][j+2], u[i][j],$
$u[i-1][j+1], u[i+1][j+1]);$ }
for $(j = 1; j < N-1; j+=2)$ {
$u[i+1][j] = f(u[i+1][j+1], u[i+1][j-1],$
$u[i][j], u[i+2][j]);$
$u[i+1][j+1] = f(u[i+1][j+2], u[i+1][j],$
$u[i][j+1], u[i+2][j+1]);$ } }
for $(j = 2; j < N-1; j+=2)$ {
$u[N-2][j] = f(u[N-2][j+1], u[N-2][j-1],$
$u[N-3][j], u[N-1][j]);$ }

A promising perspective: non-linear mapping

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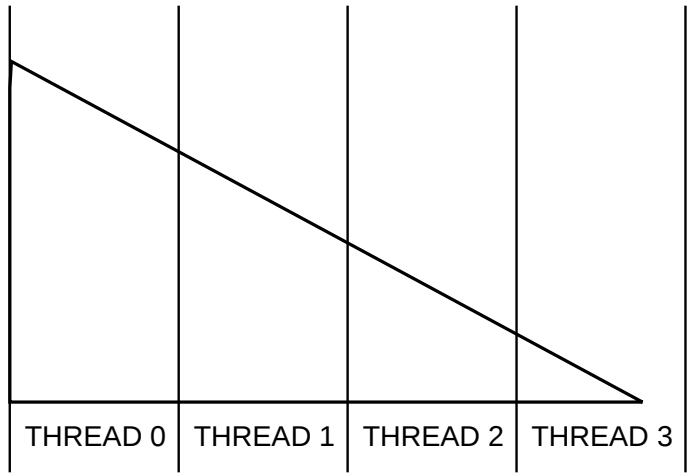
Example:

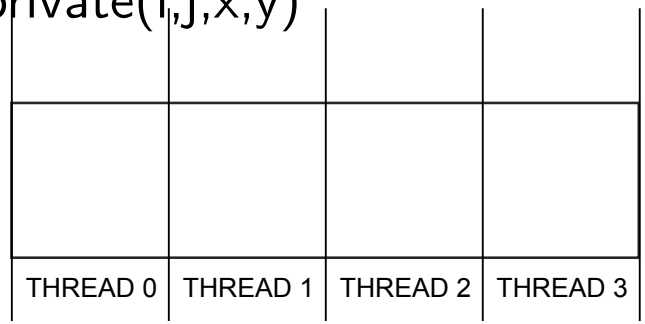
- Load unbalance

  # pragma omp parallel for shared(a,b) private(i,j)
  for $(i = 0; i < N; i++)$
  for $(j = 0; j < N - i; j++)$
  $a[j][i] = b[j][i] + 12;$



- $N(N+1)/2$ iterations
- Such a loop nest would be better:

  # pragma omp parallel for shared(a,b) private(i,j,x,y)
  for $(i = 0; i < N; i++)$
  for $(j = 0; j < (N+1)/2; j++)$
  $x = ?; y = ?;$
  $a[y][x] = b[y][x] + 12;$



A promising perspective: non-linear mapping

Example:

- Ranking polynomial of the first nest:

$$\forall (i,j) \in D_1, R_1(i,j) = Ni - \frac{i(i-1)}{2} + j + 1 = \left\{1, 2, ..., \frac{N(N+1)}{2}\right\}$$

- Ranking polynomial of the second nest:

$$\forall (i,j) \in D_2, R_2(i,j) = \frac{(N+1)}{2} i + j + 1 = \left\{1, 2, ..., \frac{N(N+1)}{2}\right\}$$

- Equation to be solved:

$$\forall (i,j) \in D_2, \exists (x,y) \in D_1 \text{ s.t. } R_1(x,y) = K = R_2(i,j)$$

A promising perspective: non-linear mapping

Example:

- Solving $R_1(x,0) - K = Ni - \frac{i(i-1)}{2} + 1 - K = 0$
- Two roots:

$$r_1 = \frac{2N + 1 - \sqrt{(2N+1)^2 + 8(1-K)}}{2}$$

$$r_2 = \frac{2N + 1 + \sqrt{(2N+1)^2 + 8(1-K)}}{2}$$

- $\lfloor r_1 \rfloor$ is the solution x of $R_1(x,y) = K$
- $\Longrightarrow\ y = K - R_1(\lfloor r_1 \rfloor, 0) = K - N\lfloor r_1 \rfloor + \frac{\lfloor r_1 \rfloor(\lfloor r_1 \rfloor - 1)}{2} - 1$

A promising perspective: non-linear mapping

Example:

- Second loop nest:

```
# pragma omp parallel for shared(a,b) private(i,j,x,y,K)
for (i = 0; i < N; i++)
 for (j = 0; j < (N + 1)/2; j++)
  K = (N + 1) * i/2 + j + 1;
  x = ((2 * N + 1) − sqrt((2 * N + 1) * (2 * N + 1) + 8 * (1 − K)))/2;
  y = K − (N * x − x * (x − 1)/2 + 1); ;
  a[y][x] = b[y][x] + 12;
```

- sqrt is very time consuming: important slow-down

A promising perspective: non-linear mapping
Example:

- New version: pre-computing a sufficient range of square roots

```
# pragma omp parallel for shared(a,b) private(i,j,x,y,K)
for (i = 0; i < N; i++)
 for (j = 0; j < (N + 1)/2; j++)
  K = (N + 1) * i/2 + j + 1;
  x = ((2 * N + 1) − tab[(2 * N + 1) * (2 * N + 1) + 8 * (1 − K)])/2;
  y = K − (N * x − x * (x − 1)/2 + 1); ;
  a[y][x] = b[y][x] + 12;
```

- 1.3 speed-up with 12 threads with the second nest vs the first ($N = 4000$, AMD Opteron 6172, 12 cores, 2.1 Ghz)

Perspectives & conclusion

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Multifor

- Many possible extensions
  - loop indices used in other loops
  - variable grain and offset
  - parallelism in several dimensions (loops, grains, offsets)
  - non-linear control
    - multiwhile?
- Inter-nests code analysis and transformations
- Implementation in CLANG-LLVM

Non-linear mapping

- Other application opportunities
  - data locality, scheduling, ...
- Non-linear analysis

THANK YOU

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