Static Analysis of OpenStream Programs

Using polyhedral techniques to analyze interesting language subsets

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Parallel languages, runtime execution, and static analysis

Solution(s) for high-level parallel programming?

- Optimizations: static or dynamic?
- Specifications: language constructs or libraries?
- Expressiveness: deterministic (no data races) or deadlock-free?
- How to represent communications and memories? Concurrency?

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Endless list of approaches:

- "Lower"-level: MPI, CUDA, OpenCL, Lime, ...
- Runtime-based: Kaapi, StarPU (with task dep. as in OpenMP 4.0), TBB, ...
- (A)PGAS languages: Co-Array Fortran, UPC, Chapel, X10, ...
- "Dataflow" languages: KPN, SDF, CSDF, StreamIt, SigmaC, OpenStream, ...
- Many other types: OpenMP, StarSs, SAC, Concurrent Collections, Galois, ...

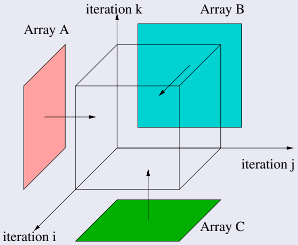
☛ Can static optimization help runtime optimizations?

Worst-case, liveness, deadlocks, races, buffer sizes, granularity, locality, ...

Multi-dimensional affine representation of loops and arrays

Matrix Multiply

```
int i,j,k;
for(i = 0; i < n; i++) {
   for(j = 0; j < n; j++) {
S:      C[i][j] = 0;
        for(k = 0; k < n; k++) {
T:          C[i][j] += A[i][k] * B[k][j];
        }
   }
}
```



Polyhedral Description — Omega/ISCC-like syntax

```
Domain := [n]->{S[i,j]: 0<=i,j<n; T[i,j,k]: 0<=i,j,k<n};

Read := [n]->{T[i,j,k]->A[i,k]; T[i,j,k]->B[k,j];
                T[i,j,k]->C[i,j]};

Write := [n]->{S[i,j]->C[i,j]; T[i,j,k]->C[i,j]};

Order := [n]->{S[i,j]->[i,j,0]; T[i,j,k]->[i,j,1,k]};
```

Triple interest of polyhedral model

Polyhedral "model", model of what?

- Specification model: affine loops, Alpha, CRP
- Provable techniques with some hypotheses: SCoP, approximations.
- Simplified form to prove hardnesss: NP-completeness, undecidability.

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Principle: study a polyhedral subset of a specification/language.

- Uniform loops as simple cases to discuss NP-completeness.
- Polyhedral X10 (Yuki, Feautrier, Rajopadhye, Saraswat, PPoPP'13).
- Polyhedral OpenStream (Pop/Cohen CDDF + this paper).

☛ Part of an effort in extending (with new techniques) and expanding (with new applications) polyhedral compilation.

Analyzing X10 through a polyhedral fragment

X10 language developed at IBM, variant at Rice (V. Sarkar)

- PGAS (partitioned global address space) memory principle.
- Parallelism of threads: in particular keywords finish, async, clock.
- No deadlocks by construction but non-determinism is possible.

Polyhedral X10 Yuki, Feautrier, Rajopadhye, Saraswat (PPoPP 2013)

Can we analyze the code for data races?

```
finish {
  for(i in 0..n-1) {
    S1;
    async {
      S2;
    }
  }
}
```

```
clocked finish {
  for(i in 0..n-1) {
    S1; advance();
    clocked async {
      S2; advance();
    }
  }
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Undecidable. Partial order $\prec_c$ defined by $\vec{x} \prec_c \vec{y}$ iff $\vec{x} \prec \vec{y}$ or $\phi(\vec{x}) < \phi(\vec{y})$. $\phi(\vec{x}) = \#$ advances before (for $\prec$) $\vec{x}$.

Analyzing OpenStream through a polyhedral fragment

```
#pragma omp task output (x) // Task T1
x = ...;

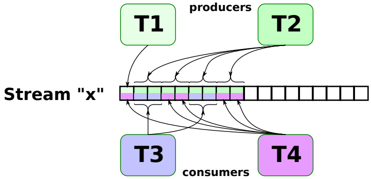
for (i = 0; i < N; ++i) {
  int window_a[2], window_b[3];

  #pragma omp task output (x « window_a[2]) // Task T2
  window_a[0] = ...; window_a[1] = ...;

  if (i % 2) {
    #pragma omp task input (x » window_b[2]) // Task T3
    use (window_b[0], window_b[1]);
  }

  #pragma omp task input (x) // Task T4
  use (x);
}
```

(Pop, Cohen, 2011)



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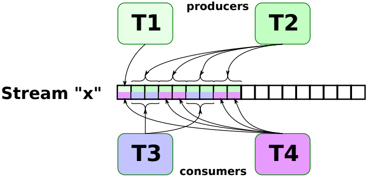

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- Unlike KPN, streams with multiple inputs/outputs (but deterministic).

- Reservation for reads/writes in streams with burst and horizon.
- Single assignment in streams (by construction) + dataflow semantics.
- The order of creations is the sequential order of the control program.

- Erbium runtime, optimizations of OpenStream explored by Pop, Miranda & Cohen. Motivates the analysis of a polyhedral fragment.

Some properties of polyhedral OpenStream

- Write/read access functions to streams are polynomials that can be expressed statically (loop counting: Ehrhart, Barvinok).

$$\text{Ex. for writes: } I_s(\vec{t}\,) = \sum_{\tau \in W_s} b_{\tau,s} \mathrm{Card}\{\vec{x} \in D_\tau \mid \vec{x} \prec_{\mathrm{lex}} \vec{t}\,\}$$

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  - Buffer of size s: window of s live elements moving to increasing indices.
- Deadlock detection is undecidable (polynomials encoding as for X10).
  - With dependences only, where a read waits for its corresponding write.
  - Even if a read must wait for all writes with smaller indices ("Kahnian").
  - Even if writes must occur in increasing order of their indices ("causal").

First ingredient (Feautrier): build multivariate polynomials

$Q(x_1, \ldots, x_n)$: multivariate polynomial, nonnegative integer coefficients.

Write:

- $Q(x) = Q(x_1, x_r)$, x_1 first variable.
- $Q^1(x_1, x_r) = Q(x_1 + 1, x_r) - Q(x_1, x_r)$ (first difference)
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- Continue with other variables:

```
phi = Q(0,x_r); // Put new loops
for (i = 0; i < x; i++) {
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Second ingredient: build the OpenStream structure

```
s, t streams;
for (x in D) {
  /* D is the n-dim. first orthant or
  the n-dim. cube of size N in it */
  R1: read Q(x) times in t;
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Deadlock situations:

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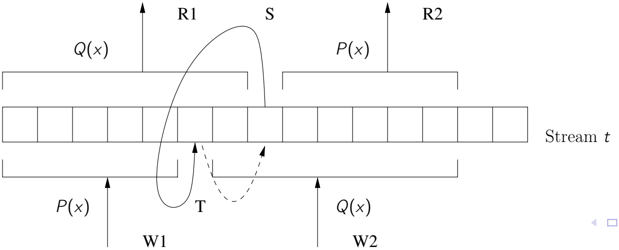

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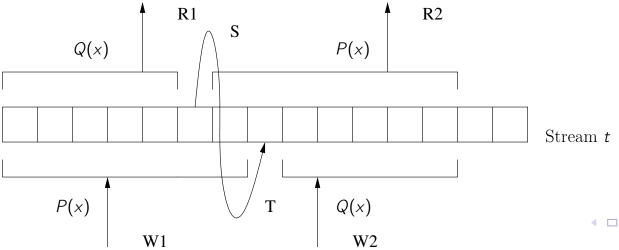

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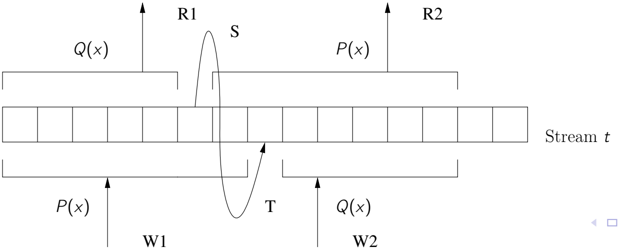
- $R(x) = 0$ iff $R^{2}(x) \leq 0$.



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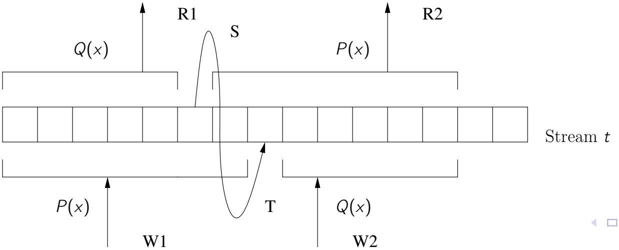

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Other problems:

- Missing producer.
- Bounded streams.

Take-home messages

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About parallel languages and their analysis/optimization

- What do you prefer: deadlocks or races?
- How to express link between user/compiler and compiler/runtime?
- Parallel constructs can help dep. analysis (e.g., Chatarasi et al. IMPACT/PACT'15).

☛ Towards the analysis of parallel languages, with better user/compiler and compiler/runtime interactions (see also next talk on liveness analysis).