Scalable Polyhedral Compilation,
Syntax vs. Semantics: 1–0 in the First Round

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Polyhedral/Affine Scheduling

(Based on the Pluto algorithm [Bondhugula et al. 2008])

Iteratively produce affine schedule functions such that:

- dependence distances are *lexicographically* positive
- dependence distances are small ⇒ temporal locality
- dependence distances are zero ⇒ parallelism
- dependences have non-negative distance along consecutive dimensions ⇒ permutability (which enables tiling)

<table>
<thead>
<tr>
<th>permutable</th>
<th>permutable</th>
<th>permutable</th>
</tr>
</thead>
<tbody>
<tr>
<td>(0, 1, 0, 0)</td>
<td>(0, 1, -2, 3)</td>
<td>(0, 0, -1, 42)</td>
</tr>
<tr>
<td>valid</td>
<td>also valid</td>
<td>violated</td>
</tr>
</tbody>
</table>
Polyhedral/Affine Scheduling

(Based on the Pluto algorithm [Bondhugula et al. 2008])

Iteratively produce affine scheduling functions of the form

\[ t_{S,k} = \vec{a} \cdot \vec{i} + \vec{b} \cdot \vec{P} + d \]

minimize \( t_{S,k} - t_{R,k} \)

for every “proximity” dependence \( R \rightarrow S \)

while enforcing dependence constraints

Statement S, scheduling step k
a,b,d – coefficients
i – original loop iterators
P – symbolic parameters
Polyhedral/Affine Scheduling

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Iteratively produce affine scheduling functions of the form

\[ t_{S,k} = \overline{a} \cdot \overline{i} + \overline{b} \cdot \overline{P} + d \]

minimize \( (t_{S,k} - t_{R,k}) \leq \overline{u} \cdot \overline{P} + w \)

for every "proximity dependence" \( R \rightarrow S \)

while enforcing dependence constraints

\[ \text{lexmin } \overline{u}, w, \overline{a}, \overline{b}, d \text{ s.t. } \overline{u} \succ 0 \]

→ Integer Linear Programming (ILP) problem
State of the Art Scheduling Algorithm Template

[Zinenko et al. CC 2018]

- Multiple notions of “proximity”, including temporal and spatial locality
- Integrate parallelization as “optional constraints”
- Iterate on two parameterizable ILP problems
  - carry as little spatial proximity relations as possible and produce coincident dimensions for parallelism (based on the Pluto algorithm [Bondhugula et al. 2008])
  - carry multiple spatial proximity relations without skewing (based on the Feautrier algorithm [Feautrier 1992])
  - play with weights and reorder dimensions in lexicographic minimization
Scalability — Principles

Challenges

- ILP, feasibility
- Projection, simplification
- Dimensionality of scheduling
- Random sampling
- Precise proximity modeling
- Precise profitability modeling

Solutions

- LP, incomplete heuristics
- Sub-polyhedral abstractions (TVPI)
- Structure and cluster statements
- Pairwise and hierarchical scheduling
- Empirical search heuristics
- Restrictions (permutations, bound coeffs)

Sub-polyhedra [Upadrasta et al. POPL 2013]
Pluto+ and LP relaxation [Acharya et al. PPoPP 2015, TOPLAS 2016, PLDI 2015]

More references in the paper
Scalability — Exposing and Exploiting Structure

isl Schedules Trees [Verdoollaeghe et al. IMPACT 2014] [Grosser et al. TOPLAS 2015]

Optimization steps for $\text{sgemm}$
Scalability — Mixing Oil and Water

isl Schedules Trees [Verdoolaege et al. IMPACT 2014] [Grosser et al. TOPLAS 2015]

Also:

Structured/modular scheduling [Feautrier IJPP 2006]

PolyAST [Shirako et al. SC 2014]
PolyMage [Mullapudi et al ASPLOS 2015]
Tensor Comprehensions [Vasilache et al. TACO 2019]
MLIR/affine https://mlir.llvm.org

This work: exploit structure by focusing on statement clustering
Clustering SCCs — “Semantics”

Clustering Strongly Connected Components (SCCs) of the reduced dependence graph
Clustering SCCs — “Semantics”

Clustering Strongly Connected Components (SCCs) of the reduced dependence graph (SCCs considering the innermost dimension only)
for (i = 0; i < N; i++)
for (j = 0; j < N; j++) {
    temp1 = A[i][j] * B[i][j];
    C[i][j] = temp1;
    temp2 = A[i][j] * C[i][j];
    D[i][j] = temp2;
}

for (i = 0; i < N; i++)
for (j = 0; j < N; j++) {
    // Macro-statement
    M0;
}

for (i = 0; i < N; i++)
for (j = 0; j < N; j++) {
    // Macro-statement
    M1;
}

Clustering basic blocks irrespectively of dependences, proximity, parallelism
Clustering — Questions

Soundness

● No cycles in the reduced dependence graph of macro statements
● Convexity of the macro statements

Completeness

● Do not miss (interesting) affine schedules
● Interaction with scheduling heuristics

Effectiveness

● Effective scalability benefits
● Effective performance results
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- Effective scalability benefits
- Effective performance results

More detail in the paper
Clustering — A Missing Experiment

Few experiment to evaluate the practical impact of clustering on scheduling effectiveness, separately from scalability

No experiment to compare different forms of clustering

- **Offline, syntax:** blocks and nesting structure in the source program, gcc/Graphite, llvm/Polly, [Mehta et a. PLDI 2015]
- **Offline, semantics:** dependence SCCs, [Meister et al. HPCS 2019]
- **Online, incremental, SCCs and proximity:** isl, [Zinenko et al. CC 2018]
- **Online, with backtracking when clustering hurts feasibility:** ?
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**Surprise:** Negative Result! Offline, syntactic does well

Caveat of the study: early experiment, considering only the Pluto optimization space, objectives and heuristics, and limited to Polybench, image processing benchmarks
Clustering — A Missing Experiment

Disclaimer... this is only a preliminary experiment...

Benchmarks

- 27 Polybench 3.2 converted to three address code (Polybench-3AC)
- 7 image processing benchmarks from the PENCIL suite
- Allen and Kennedy distribution/vectorization benchmark: “dist”
- Unconclusive experiments with SPEC and NAS from Mehta’s benchmarks

Evaluation

- PPCG 0.02 plus clustering and tweaking heuristics externally (Python)
- Dual-core x86
Scheduling Time

Median reduction in #Statements
- 2.5x for SCC
- 3x for BB
- up to 25x in some cases

Median reduction in #Deps
- 3.67x for SCC
- 4x for BB
- up to 72x in some cases
Execution Time of the Generated Code

4 optimization scenarios considered x 35 benchmarks

- SCC vs. BB clustering
- fusion vs. distribution heuristic

Identical performance, often identical code, in all but 9/150 cases

- BB clustering hurts “dist” benchmark with distribution heuristic
- Chaotic effects on statement ordering yield up to 25% difference
Early and Temporary Conclusion

Without additional effort on evaluating more advanced offline or online clustering heuristics, including more advanced schedulers, BB clustering happens to be just “good enough” (matching Polly folklore and experience)
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- IMPACT is a great venue to publish work in progress
- ... negative results
- ... and even “decremental” work!